Introduction

As we move into the second decade of the twenty-first century, one thing is clear: Our country needs highly trained workers who can wrestle with complex problems. Gone are the days when basic skills could be counted on to yield high-paying jobs and an acceptable standard of living. Especially needed are individuals who can think, reason, and engage effectively in quantitative problem solving.

The instructional practices used in the majority of our nation's classrooms will not prepare students for these new demands. National studies have shown that American students are not routinely asked to engage in conceptual thinking or complex problem solving (Stigler and Hiebert 1999). Most schoolwork consists of assignments composed of "problems" for which students have been taught a preferred method of solving. There is little engagement of student "thinking" in such tasks, only the straightforward application of previously learned skills and recall of memorized facts. It is unrealistic to expect students to learn to grapple with the unstructured, messy challenges of today's world if they are forced to sit silently in rows, complete basic skills worksheets, and engage in teacher-led "discussions" that consist of literal, fact-based questions and answers.

What kind of learning experiences *will* prepare students for the demands of the twenty-first century? *Research tells us that complex knowledge and skills are learned through social interaction* (Vygotsky 1978; Lave and Wenger 1991). We learn through a process of knowledge construction that requires us to actively manipulate and refine information and then integrate it with our prior understandings. Social interaction provides us with the opportunity to use others as resources, to share our ideas with others, and to participate in the joint construction of knowledge. In mathematics classrooms, high-quality discussions support student learning of mathematics by helping students learn how to communicate their ideas, making students' thinking public so it can be guided in mathematically sound directions, and encouraging students to evaluate their own and each other's mathematical ideas. These are all important features of what it means to be "mathematically literate."

Creating discussion-based opportunities for student learning will require learning on the part of many teachers. First, teachers will need to learn how to select and set up cognitively challenging instructional tasks in their classrooms, since such high-level tasks provide the grist for worthwhile discussions. Over the years, however, most textbooks have fed teachers a steady diet of routine, procedural tasks around which it would be difficult, if not impossible, to organize an engaging discussion.

Second, teachers must learn how to support their students as they engage with and discuss their solutions to cognitively challenging tasks. We know from our own past research that once high-level tasks are introduced in the classroom, many teachers have difficulty maintaining the cognitive demand of those tasks as students engage with them (Stein, Grover, and Henningsen 1996). Students often end up thinking and reasoning at a lower level than the task is intended to elicit. One of the reasons for this is teachers' difficulties in orchestrating discussions that productively use students' ideas and strategies that are generated in response to high-level tasks.

A typical lesson that uses a high-level instructional task proceeds in three phases. It begins with the teacher's launching of a mathematical problem that embodies important mathematical ideas and can be solved in multiple ways. During this "launch phase," the teacher introduces students to the problem, the tools that are available for working on it, and the nature of the products that the students will be expected to produce. This phase is followed by the "explore phase," in which students work on the problem, often discussing it in pairs or small groups. As students work on the problem, they are encouraged to solve it in whatever way makes sense to them and be prepared to explain their approach to others in the class. The lesson then concludes with a whole-class discussion and summary of various student-generated approaches to solving the problem. During this "discuss and summarize" phase, a variety of approaches to the problem are displayed for the whole class to view and discuss.

Why are these end-of-class discussions so difficult to orchestrate? Research tells us that students learn when they are encouraged to become the authors of their own ideas and when they are held accountable for reasoning about and understanding key ideas (Engle and Conant 2002). In practice, doing both of these simultaneously is very difficult. By their nature, high-level tasks do not lead all students to solve the problem in the same way. Rather, teachers can and should expect to see varied (both correct and incorrect) approaches to solving the task during the discussion phase of the lesson. In theory, this is a good thing because students are "authoring" (or constructing) their own ways of solving the problem.

The challenge rests in the fact that teachers must also align the many disparate approaches that students generate in response to high-level tasks with the learning goal of the lesson. It is the teachers' responsibility to move students collectively toward, and hold them accountable for, the development of a set of ideas and processes that are central to the discipline—those that are widely accepted as worthwhile and important in mathematics as well as necessary for students' future learning of mathematics in school. If the teacher fails to do this, the balance tips too far toward student authority, and classroom discussions become unmoored from accepted disciplinary understandings.

The key is to maintain the right balance. Too much focus on accountability can undermine students' authority and sense making and, unwittingly, encourage increased reliance on teacher direction. Students quickly get the message—often from subtle cues—that "knowing mathematics" means using only those strategies that have been validated by the teacher or textbook; correspondingly, they learn not to use or trust their own reasoning. Too much focus on student authorship, on the other hand, leads to classroom discussions that are free-for-alls.

Successful or Superficial? Discussion in David Crane's Classroom

In short, the teacher's role in discussions is critical. Without expert guidance, discussions in mathematics classrooms can easily devolve into the teacher taking over the lesson and providing a "lecture," on the one hand, or, on the other, the students presenting an unconnected series of show-and-tell demonstrations, all of which are treated equally and together illuminate little about the mathematical ideas that are the goal of the lesson. Consider, for example, the following vignette (from Stein and colleagues [2008]), featuring a fourth-grade teacher, David Crane.

ACTIVE ENGAGEMENT 0.1

As you read the Case of David Crane, identify instances of student authorship of ideas and approaches, as well as instances of holding students accountable to the discipline.

Leaves and Caterpillars: The Case of David Crane

Students in Mr. Crane's fourth-grade class were solving the following problem: "A fourth-grade class needs 5 leaves each day to feed its 2 caterpillars. How many leaves would the students need each day for 12 caterpillars?" Mr. Crane told his students that they could solve the problem any way they wanted, but he emphasized that they needed to be able to explain how they got their answer and why it worked.

As students worked in pairs to solve the problem, Mr. Crane walked around the room, making sure that students were on task and making progress on the problem. He was pleased to see that students were using many different approaches to the problem—making tables, drawing pictures, and, in some cases, writing explanations.

He noticed that two pairs of students had gotten wrong answers (see fig. 0.1). Mr. Crane wasn't too concerned about the incorrect responses, however, since he felt that once several correct solution strategies were presented, these students would see what they did wrong and have new strategies for solving similar problems in the future.

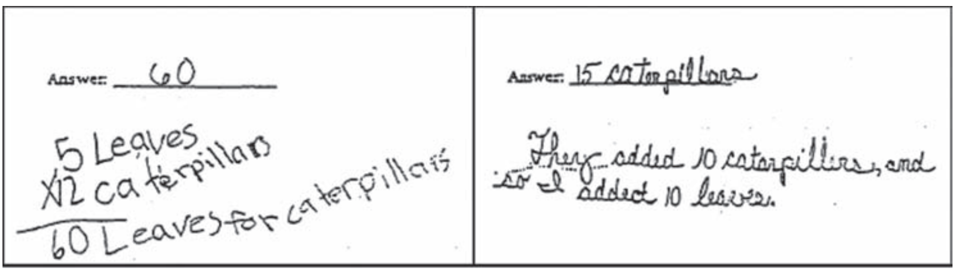

Fig. 0.1. Solutions produced by Darnell and Marcus (left) and Missy and Kate (right)

When most students were finished, Mr. Crane called the class together to discuss the problem. He began the discussion by asking for volunteers to share their solutions and strategies, being careful to avoid calling on the students with incorrect solutions. Over the course of the next 15 minutes, first Kyra, then Jason, Jamal, Melissa, Martin, and Janine volunteered to present the solutions to the task that they and their partners had created (see fig. 0.2). During each presentation, Mr. Crane made sure to ask each presenter questions that helped the student to clarify and justify the work. He concluded the class by telling students that the problem could be solved in many different ways and now, when they solved a problem like this, they could pick the way they liked best because all the ways gave the same answer.

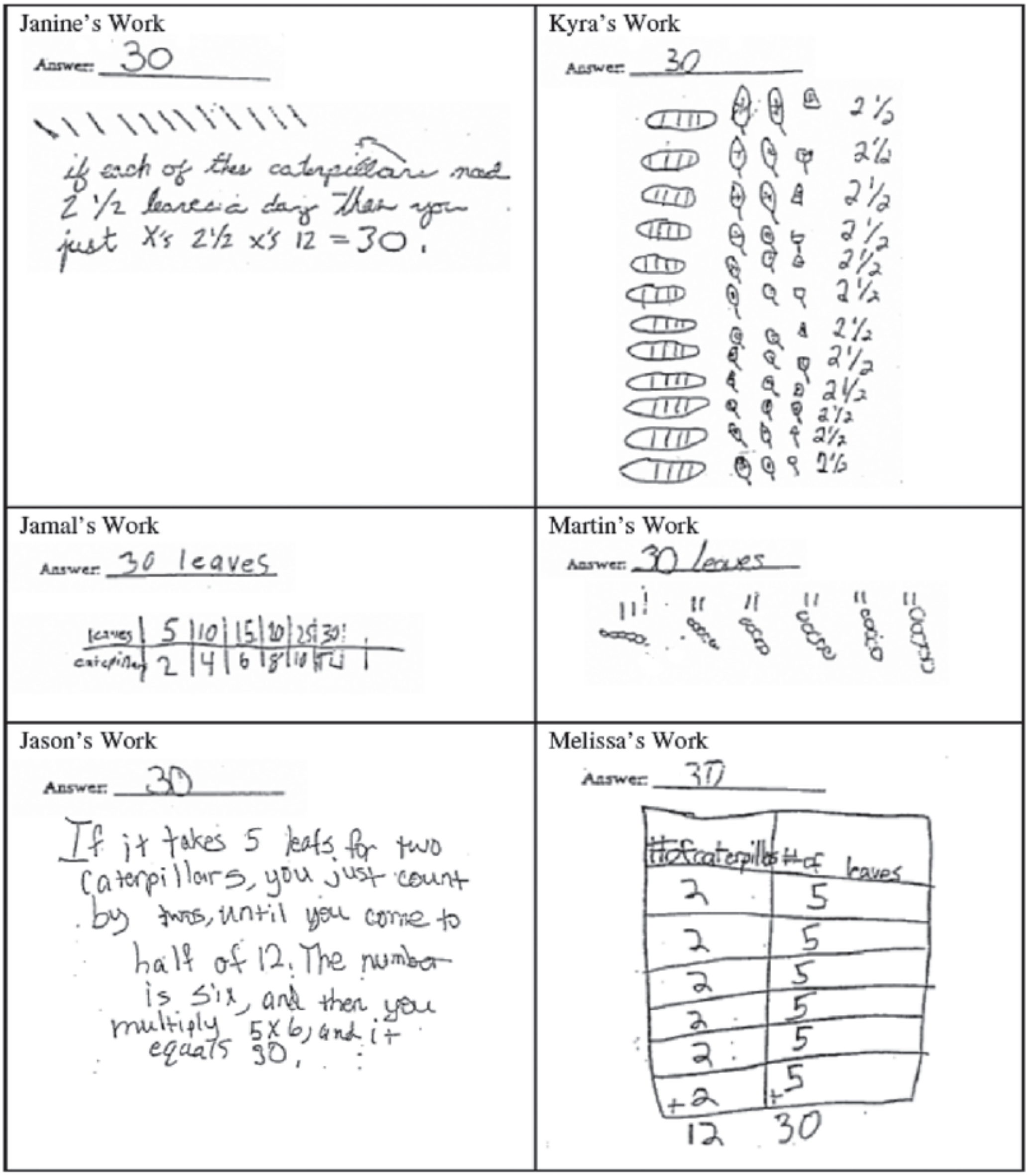

Fig. 0.2. Solutions shared by students in Mr. Crane's class

Analyzing the Case of David Crane

Some would consider Mr. Crane's lesson exemplary. Indeed, Mr. Crane did many things well, including allowing students to construct their own way of solving this cognitively challenging task and stressing the importance of students' being able to explain their reasoning. Students were working with partners and publicly sharing their solutions and strategies with their peers; their ideas appeared to be respected. All in all, students in Mr. Crane's class had the opportunity to become the "authors" of their own knowledge of mathematics.

However, a more critical eye might have noted that the string of presentations did not build toward important mathematical ideas. The upshot of the discussion appeared to be "the more ways of solving the problem, the better," but, in fact, Mr. Crane held each student accountable for knowing only one way to solve the problem. In addition, although Mr. Crane observed students as they worked, he did not appear to use this time to assess what students understood about proportional reasoning or to select particular students' work to feature in the whole-class discussion. Furthermore, he gathered no information regarding whether the two pairs of students who had gotten the wrong answer (Darnell and Marcus, and Missy and Kate) were helped by the student presentations of correct strategies. Had they diagnosed the faulty reasoning in their approaches?

In fact, we argue that much of the discussion in Mr. Crane's classroom was show-and-tell, in which students with correct answers each take turns sharing their solution strategies. The teacher did little filtering of the mathematical ideas that each strategy helped to illustrate, nor did he make any attempt to highlight those ideas. In addition, the teacher did not draw connections among different solution methods or tie them to important disciplinary methods or mathematical ideas. Finally, he gave no attention to weighing which strategies might be most useful, efficient, accurate, and so on, in particular circumstances. All were treated as equally good.

In short, providing students with cognitively demanding tasks with which to engage and then conducting show-and-tell discussions cannot be counted on to move an entire class forward mathematically. Indeed, this kind of practice has been criticized for creating classroom environments in which nearly complete control of the mathematical agenda is relinquished to students. Some teachers misperceived the appeal to honor students' thinking and reasoning as a call for a complete moratorium on teachers' shaping of the quality of students' mathematical thinking. As a result of the lack of guidance with respect to what teachers *could* do to encourage rigorous mathematical thinking and reasoning, many teachers were left feeling that they should avoid telling students anything.

A related criticism of inquiry-oriented lessons concerns the fragmented and often incoherent nature of the discuss-and-summarize phases of lessons. In these show-and-tells, as exemplified in David Crane's classroom, one student presentation would follow another with limited teacher (or student) commentary and no assistance with respect to drawing connections among the methods or tying them to widely shared disciplinary methods and concepts. The discussion offered no mathematical or other reason for students to necessarily listen to or try to understand the methods of their classmates. As illustrated in Mr. Crane's comment at the end of the class, students could simply "pick the way they liked best." This type of situation has led to an increasingly recognized dilemma associated with inquiry- and discovery-based approaches to teaching: the challenge of aligning students' developing ideas and methods with the disciplinary ideas that they ultimately are accountable for knowing.

In sum, David Crane did little to encourage accountability to the discipline of mathematics. How could he have more firmly supported student accountability without undermining student authority? The single most important thing that he could have done would be to have set a clear goal for what he wanted students to learn from the lesson. Without a learning objective in mind, the various solutions that were presented, although all correct, were scattered in the "mathematical landscape." If, however, he had targeted the learning goal of, for example, making sure that all students recognized that the relationship between caterpillars and leaves was multiplicative and not additive, he might have monitored students' work with this in mind. Whose work illustrated the multiplicative relationship particularly well? Did the students' work include examples of different ways of illustrating this relationship—examples that could connect with known mathematical strategies (e.g., unit rate, scaling up)? This assessment of student work would have allowed him to be more deliberate about which students he selected to present during the discussion phase. He might even have wanted to have the incorrect, additive solutions displayed so that students could recognize the faulty reasoning that underlie them. With an array of purposefully selected strategies presented, Mr. Crane would then be in a position to steer the discussion toward a more mathematically satisfying conclusion.

Conclusion

The Case of David Crane illustrates the need for guidance in shaping classroom discussions and maximizing their potential to extend students' thinking and connect it to important mathematical ideas. The chapters that follow offer this guidance by elaborating a practical framework, based on five doable instructional practices, for orchestrating and managing productive classroom discussions.

CHAPTER 1

Introducing the Five Practices

Many teachers are daunted by an approach to pedagogy that builds on student thinking. Some are worried about content coverage, asking, "How can I be assured that students will learn what I am responsible for teaching if I don't march through the material and tell them everything they need to know?" Others—teachers who perhaps are already convinced of the importance of student thinking—may be nonetheless worried about their ability to diagnose students' thinking on the fly and to quickly devise responses that will guide students to the correct mathematical understanding.

Teachers are correct when they acknowledge that this type of teaching is demanding. It requires knowledge of the relevant mathematical content, of student thinking about that content, and of the subtle pedagogical "moves" that a teacher can make to lead discussions in fruitful directions, along with the ability to rapidly apply all of this in specific circumstances. Yet, we have seen many teachers learn to teach in this way, with the help of the five practices.

We think of the five practices as skillful improvisation. The practices that we have identified are meant to make student-centered instruction more manageable by moderating the degree of improvisation required by the teacher during a discussion. Instead of focusing on in-the-moment responses to student contributions, the practices emphasize the importance of planning. Through planning, teachers can anticipate likely student contributions, prepare responses that they might make to them, and make decisions about how to structure students' presentations to further their mathematical agenda for the lesson. We turn now to an explication of the five practices.

The Five Practices

The five practices were designed to help teachers to use students' responses to advance the mathematical understanding of the class as a whole by providing teachers with some control over what is likely to happen in the discussion as well as more time to make instructional decisions by shifting some of the decision making to the planning phase of the lesson. The five practices are—

1. ***anticipating*** likely student responses to challenging mathematical tasks;
2. ***monitoring*** students' actual responses to the tasks (while students work on the tasks in pairs or small groups);
3. ***selecting*** particular students to present their mathematical work during the whole-class discussion;
4. ***sequencing*** the student responses that will be displayed in a specific order; and
5. ***connecting*** different students' responses and connecting the responses to key mathematical ideas.

Each practice is described in more detail in the following sections, which illustrate them by identifying what Mr. Crane *could have done* in the Leaves and Caterpillars lesson (presented in the introduction), to move student thinking more skillfully toward the goal of recognizing that the relationship between caterpillars and leaves is multiplicative, not additive.

Anticipating

The first practice is to make an effort to actively envision how students might mathematically approach the instructional task or tasks that they will work on. This involves much more than simply evaluating whether a task is at the right level of difficulty or of sufficient interest to students, and it goes beyond considering whether or not they are getting the "right" answer.

Anticipating students' responses involves developing considered expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—that they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn.

Anticipating requires that teachers do the problem as many ways as they can. Sometimes teachers find that it is helpful to expand on what they might be able to think of individually by working on the task with colleagues, reviewing responses to the task that might be available (e.g., work produced by students in the previous year, responses that are published along with tasks in supplementary materials), and consulting research on student learning of the mathematical ideas embedded in the task. For example, research suggests that students often use additive strategies (such Missy and Kate's response, shown in fig. 0.1) to solve tasks like the Leaves and Caterpillars problem, in which there is a multiplicative relationship between quantities (Hart 1981; Heller et al. 1989; Kaput and West 1994). Anticipating this approach in advance of the lesson would have made it possible for Mr. Crane to recognize it when his students produced it and carefully consider what actions he might take should they do so (e.g., what questions to ask so that students become aware of the multiplicative nature of the relationship between the caterpillars and leaves, how to bring up the solution during discussion so that all students might consider why it is not a valid method).

In addition, if Mr. Crane had solved the problem ahead of time in as many ways as possible, he might have realized that there were at least two different strategies for arriving at the correct answer—unit rate and scale factor—and that each of these could be expressed with different representations (pictures, tables, and written explanations).

Monitoring

Monitoring student responses involves paying close attention to students' mathematical thinking and solution strategies as they work on the task. Teachers generally do this by circulating around the classroom while students work either individually or in small groups. Carefully attending to what students do as they work makes it possible for teachers to use their observations to decide what and whom to focus on during the discussion that follows (Lampert 2001).

One way to facilitate the monitoring process is for the teacher, before beginning the lesson, to create a list of solutions that he or she anticipates that students will produce and that will help in accomplishing his or her mathematical goals for the lesson. The list, such as the one shown in column 1 of the chart in figure 1.1 for the Leaves and Caterpillars task, can help the teacher keep track of which students or groups produced which solutions or brought out which ideas that he or she wants to make sure to capture during the whole-group discussion. The "Other" cell in the first column provides the teacher with the opportunity to capture ideas that he or she had *not* anticipated.

Strategy	Who and What	Order
Unit rate Find the number of leaves eaten by one caterpillar (2.5) and multiply by 12 or add the amount for one 12 times	Janine – multiplied 12 × 2.5 (sticks representing caterpillars) Kyra – added 2.5 12 times (picture of leaves and caterpillars)	
Scale Factor Find that the number of caterpillars (12) is 6 times the original amount (2), so the number of leaves (30) must be 6 times the original amount (5)	Jason – narrative description	
Scaling Up Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillars, until you reach the desired number of caterpillars	Jamal – table with leaves and caterpillars increasing in increments of 2 and 5	
Additive Find that the number of caterpillars has increased by 10 (2 + 10 = 12), so the number of leaves must also increase by 10 (5 + 10 = 15)	Missy and Kate	
Other Scaling up by collecting sets of 2 leaves and 5 caterpillars	Martin (picture) Melissa (table)	

Fig. 1.1. A chart for monitoring students' work on the Leaves and Caterpillars task

As discussed in the introduction, Mr. Crane's lesson provided limited, if any, evidence of active monitoring. Although Mr. Crane knew who got correct answers and who did not and that a range of strategies had been used, his choice of students to present at the end of the class suggests that he had not monitored the specific mathematical learning potential available in any of the responses. What Mr. Crane could have done while students worked on the task is shown in the second column in the chart in figure 1.1.

It is important to note, however, that monitoring involves more than just watching and listening to students. During this time, the teacher should also ask questions that will make students' thinking visible, help students clarify their thinking, ensure that members of the group are all engaged in the activity, and press students to consider aspects of the task to which they need to attend. Many of these questions can be planned in advance of the lesson, on the basis of the anticipated solutions. For example, if Mr. Crane had anticipated that a student would use a unit-rate approach (Janine's or Kyra's responses—see fig. 1.2), reasoning from the fact that the number of leaves eaten by one caterpillar was 2.5, then he might have been prepared to question, say, for example, Janine, regarding how she came up with the number 2.5 and how she knew to multiply it by 12. Questioning a student or group of students while they are exploring the task provides them with the opportunity to refine or revise their strategy prior to whole-group discussion and provides the teacher with insights regarding what the student understands about the problem and the mathematical ideas embedded in it.

Selecting

Having monitored the available student strategies in the class, the teacher can then select particular students to share their work with the rest of the class to get specific mathematics into the open for examination, thus giving the teacher more control over the discussion (Lampert 2001). The selection of particular students and their solutions is guided by the mathematical goal for the lesson and the teacher's assessment of how each contribution will contribute to that goal. Thus, the teacher selects certain students to present because of the mathematics in their responses.

A typical way to accomplish "selection" is to call on specific students (or groups of students) to present their work as the discussion proceeds. Alternatively, the teacher may let students know before the discussion that they will be presenting their work. In a hybrid variety, a teacher might ask for volunteers but then select a particular student that he or she knows is one of several who have a particularly useful idea to share with the class. By calling for volunteers but then strategically selecting from among them, the teacher signals appreciation for students' spontaneous contributions, while at the same time keeping control of the ideas that are publicly presented.

Returning to the Leaves and Caterpillar vignette, if we look at the strategies that were shared, we note that Kyra and Janine had similar strategies that used the idea of unit rate (i.e., finding out the number of leaves needed for one caterpillar). Given that, there may not have been any added mathematical value to sharing both. In fact, if Mr. Crane wanted to students to see the multiplicative nature of the relationship, he might have selected Janine, since her approach clearly involved multiplication.

Also, there might have been some payoff from sharing the solution produced by Missy and Kate (fig. 0.1) and contrasting it with the solution produced by Melissa (fig. 0.2). Although both approaches used addition, Missy and Kate inappropriately added the same number (10) to both the leaves and the caterpillars. Melissa, on the other hand, added 5 leaves for every 2 caterpillars, illustrating that she understood that this ratio (5 for every 2) had to be kept constant.

Sequencing

Having selected particular students to present, the teacher can then make decisions regarding how to sequence the student presentations. By making purposeful choices about the order in which students' work is shared, teachers can maximize the chances of achieving their mathematical goals for the discussion. For example, the teacher might want to have the strategy used by the majority of students presented before those that only a few students used, to validate the work that the majority of students did and make the beginning of the discussion accessible to as many students as possible. Alternatively, the teacher might want to begin with a strategy that is more concrete (using drawings or concrete materials) and move to strategies that are more abstract (using algebra). This approach—moving from concrete to abstract—serves to validate less sophisticated approaches and allows for connections among approaches. If a common misconception underlies a strategy that several students used, the teacher might want to have it addressed first so that the class can clear up that misunderstanding to be able to work on developing more successful ways of tackling the problem. Finally, the teacher might want to have related or contrasting strategies presented one right after the other in order to make it easier for the class to compare them. Again, during planning the teacher can consider possible ways of sequencing anticipated responses to highlight the mathematical ideas that are key to the lesson. Unanticipated responses can then be fitted into the sequence as the teacher makes final decisions about what is going to be presented.

More research needs to be done to compare the value of different sequencing methods, but we want to emphasize here that particular sequences can be used to advance particular goals for a lesson. Returning to the Leaves and Caterpillar vignette, we point out one sequence that could have been used: Martin (scaling up by collecting sets—picture), Jamal (scaling up—table), Janine (unit rate—picture/written explanation); and Jason (scale factor—written explanation).

This ordering begins with the least sophisticated representation (a picture) of the least sophisticated strategy (scaling up by collecting sets) and ends with the most sophisticated strategy (scale factor), a sequencing that would help with the goal of accessibility. In addition, by having the same strategy (scaling up) embodied in two different representations (a picture and a table), students would have the opportunity to develop deeper understandings of how to think about this problem in terms of scaling up.

Connecting

Finally, the teacher helps students draw connections between their solutions and other students' solutions as well as the key mathematical ideas in the lesson. The teacher can help students to make judgments about the consequences of different approaches for the range of problems that can be solved, one's likely accuracy and efficiency in solving them, and the kinds of mathematical patterns that can be most easily discerned. Rather than having mathematical discussions consist of separate presentations of different ways to solve a particular problem, the goal is to have student presentations build on one another to develop powerful mathematical ideas.

Returning to Mr. Crane's class, let's suppose that the sequencing of student presentations was Martin, Jamal, Janine, and Jason, as discussed above. Students could be asked to compare Jamal and Janine's responses and to identify where Janine's unit rate (2.5 leaves per caterpillar) is found in Jamal's table (it is the factor by which the number of caterpillars must be multiplied to get the number of leaves). Students could also be asked to compare Jason's explanation with Jamal and Martin's work to see if the scale factor of 6 can be seen in each of their tabular and pictorial representations.

It is important to note that the five practices build on another. Monitoring is less daunting if the teacher has taken the time to anticipate ways in which students might solve a task. Although a teacher cannot know with 100 percent certainty how students will solve a problem prior to the lesson, many solutions can be anticipated and thus easily recognized during monitoring. A teacher who has already thought about the mathematics represented by those solutions can turn his or her attention to making mathematical sense of those solutions that are unanticipated. Selecting, sequencing, and connecting, in turn, build on effective monitoring. Effective monitoring will yield the substance for a discussion that builds on student thinking, yet moves assuredly toward the mathematical goal of the lesson.

Conclusion

The purpose of the five practices is to provide teachers with more control over student-centered pedagogy. They do so by allowing the teacher to manage the content that will be discussed and how it will be discussed. Through careful planning, the amount of improvisation required by the teacher "in the moment" is kept to a minimum. Thus, teachers are freed up to listen to and make sense of outlier strategies and to thoughtfully plan connections between different ways of solving problems. All of this leads to more coherent, yet student-focused, discussions.

In the next chapter, we explore an important first step in enacting the five practices: setting goals for instruction and identifying appropriate tasks. Although this work is not one of the five practices, it is the foundation on which the five practices are built. In chapters 3, 4, and 5, we then explore the five practices in depth and provide additional illustrations showing what the practices look like when enacted and how the practices can lead to more productive discussions.